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A new formulation for radiation from open-ended guiding structures is developed and applied to open-ended transmission lines, coaxial cables, parallel-plate and circular waveguides. The theory is also extended to flared radiators.

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Giorgio Franceschetti

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PREFACE

The research contained in this report was begun by Professor Franceschetti in the Spring of 1976, and a presentation of some preliminary results was given by him at the National Conference on Electromagnetic Scattering held in Chicago in June 1976 (see Communications Laboratory Report 76-1).

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March 1977.

Piergiorgio L.E. Uslenghi
Director of the Laboratory

TABLE OF CONTENTS

Chapter		Page
	PREFACE	ii
	TABLE OF CONTENTS	iii
	LIST OF FIGURES	iv
	ABSTRACT	1
1	THE PROBLEM	2
2	RELEVANT FORMULAS AND EXPANSIONS	7
3	WEINSTEIN'S SOLUTION FOR THE PARALLEL PLATE OPEN-ENDED WAVEGUIDE	11
4	THE ELEMENTARY SCATTERING COEFFICIENTS	18
5	RADIATION FROM OPEN-ENDED CIRCULAR WAVEGUIDE: AZYMUTHALLY SYMMETRIC MODES	21
6	RADIATION FROM OPEN-ENDED CIRCULAR WAVEGUIDES CONTINUED: AZYMUTHALLY ASYMMETRIC MODES	25
7	RADIATION FROM OTHER OPEN-ENDED GUIDING STRUCTURES	29
8	THE INCLUSION OF A FLARE ANGLE	31
9	CONCLUDING REMARKS	35
	REFERENCES	37

LIST OF FIGURES

Figure Page

1. Radiation from a wire antenna....................................3
2. Radiation from a two-wire transmission line........................5
3. Radiation from an open-ended parallel plate waveguide............11
4. Ray tracing inside a parallel-plate waveguide....................15
5. Ray optical description of the interaction process................17
6. Geometry of elementary radiation...............................19
7. Radiation from an open-ended circular waveguide..................21
8. Radiation from open-ended flared parallel-plate waveguide.........31

ABSTRACT

A new formulation for radiation from open-ended guiding structures is presented. The differences between this formulation and conventional physical optics or diffraction ray theory is discussed. The theory has quite general validity and is here applied in detail to open-ended transmission lines, coaxial cables, parallel plate waveguides and circular waveguides. For the examples discussed, checking with existing solutions is provided. Finally, the theory is extended to the case of flared open-ended guides.

CHAPTER I

THE PROBLEM

Radiation from apertures is certainly important from the application's viewpoint. In this paper radiation from open-ended guiding structures, as waveguides, coaxial-cables and horns, is considered. The classical way of analysis is based on the so-called *aperture method* [1] which uses the physical optics approach coupled with Kirchhoff's approximation. More recently, optical or *quasi-optical* techniques have been applied, decomposing the wave inside the guide into a set of plane waves and then considering the scattering of these plane waves by the edges of the guide. The basic idea is to model the latter problem as the scattering of a plane wave by a metal half-plane. These ideas and tools have been applied to the analysis of reflection inside [2-7], and radiation from the guide [8-17]. A different model has also been suggested for the analysis of the scattering process: a half-plane over either an electric or a magnetic infinite wall [18-19]. It is also important to recognize that exact solutions to the radiation problem are available in a limited number of cases: the parallel-plate and cylindrical waveguides [20], the latter with either azymutally symmetric or asymmetric excitation. The scattering by two half-planes has also been considered [21].

The aperture method has well known limitations and shortcomings. It is reasonably simple only when the scalar approximation is adopted; the vector theory, however, yields only marginal improvements [11].

The quasi-optical approach seems to give good results. At transitional regions, e.g., at lit-shadow boundaries, the expression of the diffraction coefficient becomes rather involved [22, 23]. In the caustic regions, either fictitious equivalent edge currents [24] or a spectral theory [25] should be adopted; the latter is also useful, if not necessary, for non-planar excitation. In the case of multiple reflections, as always happens for apertures, the analysis although straightforward in principle, is extremely cumbersome in practice.

There is, however, at least another possibility for computing radiation from open-ended guiding structures. Probably the best way to introduce the basic philosophy of the new method is to consider a wire antenna with sinusoidal current distribution $I_o \sin\beta_a z$ (see Fig. 1) and a time dependence $\exp(i\omega t)$. The far-field phasors can be easily computed, the result being:

$$\underline{E} = \zeta \frac{I_o \sin\Theta}{4\pi r} \hat{\Theta} \exp(-i\beta r) \frac{i\cos\Theta_a}{\cos^2\Theta - \cos^2\Theta_a} + \exp(-i\beta\cos\Theta) \frac{\cos\Theta\sin\beta_a l + \cos\Theta_a\cos\beta_a l}{\cos^2\Theta - \cos^2\Theta_a} \quad (1.1)$$

$$\underline{H} = \frac{1}{\zeta} E_\Theta \hat{\phi} , \qquad \zeta = \sqrt{\mu/\varepsilon}, \quad \beta = \omega\sqrt{\varepsilon\mu} \quad (1.2)$$

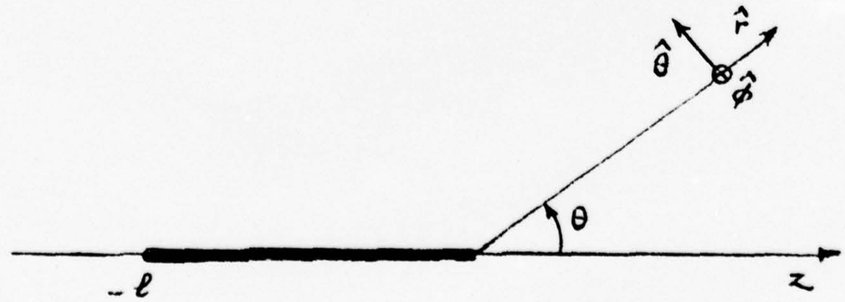

Fig. 1. Radiation from a wire antenna.

where $\beta_a = \beta\cos\Theta_a$.

Eq. (1.1) suggests this model for the radiation process: the far-field is given as the superposition of two spherical waves from the end-points $z = 0; -l.$ of the antenna. For a wire infinitely long in the $-z$ direction the second term in the bracket of (1.1) can be neglected, and we have:

$$\begin{cases} E_\Theta = \zeta \dfrac{I_i}{2\pi r} \dfrac{\sin\Theta\cos\Theta_a}{\cos\Theta + \cos\Theta_a} \dfrac{\exp(-i\beta r)}{\cos\Theta - \cos\Theta_a} & (1.3) \\ \\ H_\phi = E_\Theta/\zeta & (1.4) \end{cases}$$

where $I_i = - I_o/2i$ is the amplitude of the *incident current.* Note that the electromagnetic field (1.3-4) can be conveniently derived from a vector potential

$$\underline{A} = -\zeta \frac{I_i}{-i\omega} \frac{1}{2\pi r} \frac{\cos\Theta_a}{\cos\Theta + \cos\Theta_a} \frac{\exp(-i\beta r)}{\cos\Theta - \cos\Theta_a} \hat{z} \tag{1.5}$$

wherein the connection with far-fields is given by

$$\begin{cases} \underline{E} = -i\omega\, \hat{r} \times \underline{A} \times \hat{r} & (1.6) \\ \\ \underline{H} = -\dfrac{i\omega}{\zeta}\, \hat{r} \times \underline{A} & (1.7) \end{cases}$$

For a two-wire transmission line--the simplest conceivable guiding structure--, TEM assumption for the incident wave ($\Theta_a = 0$), hypothesis of no higher order interaction between the two wires, and superposition give (see Fig. 2):

$$\underline{A} = \zeta \frac{I_i}{-i\omega} \frac{\exp(-i\beta r)}{2\pi r} \frac{\exp(i\beta a\sin\Theta\cos\phi) - \exp(-i\beta a\sin\Theta\cos\phi)}{1 - \sin^2\Theta\sin^2\phi} \hat{y}$$

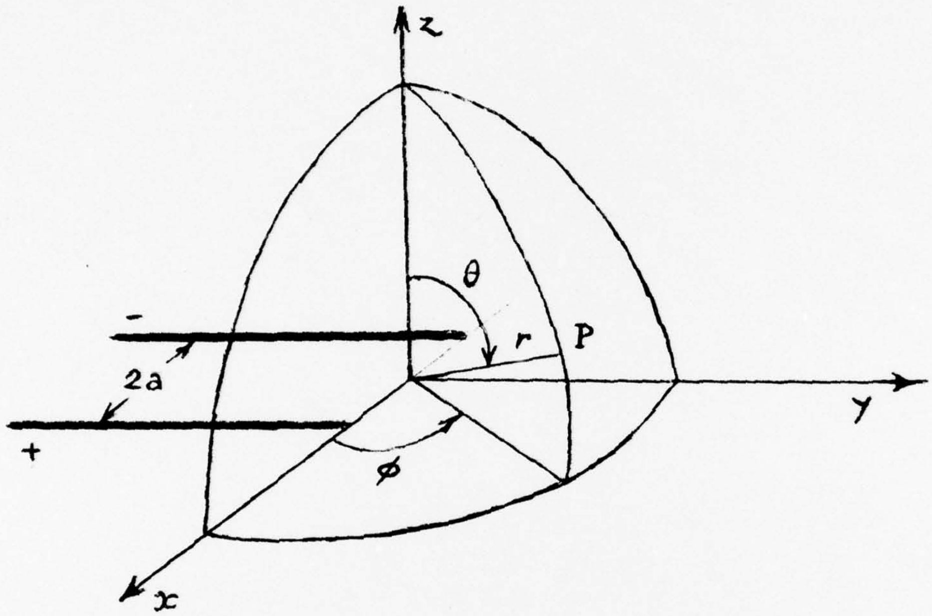

Fig. 2. Radiation from a two-wire transmission line.

Note that the field is everywhere well-behaved. In particular the (normalized) radiation diagram is given by:

$$\text{E-plane} \quad (\Theta = 90^o) \quad : \quad \frac{\sin(\beta a\cos\phi)}{\beta a\cos\phi} \tag{1.5}$$

$$\text{H-plane} \quad (\phi = 0) \quad : \quad \sin(\beta a\sin\Theta) \tag{1.6}$$

The radiation process is modelled as two spherical waves from the two end-points of the transmission line. This model has a simple physical counterpart in time-domain when a pulsed voltage is propagating along the transmission line. The local charges induced on the two curves (apparently) move with constant velocity and radiation takes place only at the very end of the line where charges are reflected with a (discontinuous) change of velocity direction. This idea has been exploited in [26]. Note further, for future reference, that the fields individually radiated by each wire diverge in the direction $\Theta = 0$, i.e., the direction of the incident wave. However, due to the dispersion equation of the line, $\cos\Theta_a = 1$, the two fields compensate and produce a finite result along $\Theta = 0$.

For an open-ended guiding structure we will have surface currents incident on the truncated rim of the guide. Accordingly, the *radiation process can be modelled as superposition of elemetary spherical waves coming from each point of the rim.* Obviously, a satisfactory theory should take into account interaction between incident surface currents. We will derive radiation coefficients associated with incident surface currents from the simplest possible canonical problem for which an exact solution exists: the parallel-plate open-ended waveguide.

It is important to recognize the profound difference between this approach and the quasi-optical one. The latter emphasizing the geometrical relationship between field-point and rim-point, so that the radiated field is due to a finite number of contributions along the rim (except in caustic regions where, in any case, the quasi-optical approach fails to give a satisfactory answer). On the contrary, this new approach is still based on a field theory rather than a ray theory and the radiation is computed as an integral along the truncated rim. Accordingly, as long as the radiation coefficients are well-behaved, no problem can be anticipated for the computed field everywhere. It is here noted that this method has nothing to do with the equivalent edge current procedure [24], which is still based on a ray-theory model and makes reference to fictitious electric and magnetic currents.

CHAPTER 2

RELEVANT FORMULAS AND EXPANSIONS

For the reader's convenience, most of the mathematical formulas used in the next sections are collected below.

Weinstein's diffraction function, exact [27].

Definition:

$$U^{+}(\sigma, \rho) = \frac{-1}{2\pi i}\int_{-\infty}^{+\infty} \ln[1 \pm \exp(-i\rho - \rho u^2)] \frac{(1 - iu^2)(1 - i\frac{u^2}{2})^{1/2}}{u(1 - i\frac{u^2}{2})^{1/2} - \frac{1 - i}{2}\sigma} du \qquad (2.1)$$

Properties:

$$U^{\pm}(-\sigma, \rho) = -U^{\pm}(\sigma, \rho) \qquad (2.2)$$

$$U^{\pm}(\pm 0, \rho) = \pm \frac{1}{2} \ln[1 \pm \exp(-i\rho)] \qquad (2.3)$$

Expansion for large σ and ρ:

$$U^{\pm}(\sigma, \rho) = \frac{-\exp(-i\pi/4)}{\sqrt{2\pi\rho}\;\sigma} \sum_{1}^{\infty}{}_{n} \frac{(\mp)^n \exp(-in\rho)}{n^{3/2}} + 0\,[\sigma^{-3}\rho^{-3/2}] \qquad (2.4)$$

Expansion for small ρ:

$$U^{+}(\sigma, \rho) = \frac{1}{2} \ln[1 + \exp(-i\rho)] + \frac{i\rho\sigma}{2\pi} [\ln \frac{4\pi}{\rho} + 1 - \gamma - \frac{\pi i}{2}] + 0\,[\rho\sigma] \qquad (2.4)$$

$$U^{-}(\sigma, \rho) = \frac{1}{2} \ln[\rho(1 + \sigma)] + \frac{\pi i}{4} - \frac{i\rho\sigma}{2\pi} [\ln \frac{4\pi}{\rho} + 1 - \gamma - \frac{\pi i}{2}] + 0\,[\rho\sigma] \qquad (2.5)$$

Weinstein's diffraction function, approximated [20, 27].

For large ρ the main contribution to the integral (2.1) arises from the vicinity of the saddle point $u = 0$. Thus, neglecting all terms of order u^2 [27]:

$$\overline{U}^{\pm}(\sigma, \rho) = \frac{-1}{2\pi i} \int_{-\infty}^{+\infty} \ln[1 \pm \exp(-i\rho - \rho u^2)] \quad \frac{du}{u - \frac{1-i}{2}\sigma} \tag{2.6}$$

The difference between $U^{\pm}$ and $\overline{U}^{\pm}$ is of $O[\rho^{-1}]$ uniformly over the range $-1 \leq \sigma \leq 1$. Letting $\rho = 2\pi q$ and $\sqrt{\rho}\,\sigma = s$ (and using the same symbol $U^{\pm}$):

$$U^{\pm}(s, q) = \frac{-1}{2\pi i} \int_{-\infty}^{+\infty} \ln[1 \pm \exp(-2\pi i q - \frac{u^2}{2})] \frac{du}{u - (1 - i)s} \tag{2.7}$$

which is the expression given in [20]. Tables of (the complex conjugate of) U^- are referred to in [20, 28].

Properties:

Same as (2.2-3)

$$U^{\pm}(s, q + n) = U^{\pm}(s, q) \tag{2.8}$$

$$U^{+}(s, q) = U^{-}(s, q + \frac{1}{2}) \tag{2.9}$$

Expansion for large s:

$$U^{-}(s, q) = -\frac{\exp(-i\pi/4)}{\sqrt{2\pi}\, s} \sum_{0\ m}^{\infty} \frac{(2m - 1)!!}{[s\exp(-i\pi/4)]^{2m}} \sum_{0\ n}^{\infty} \frac{\exp(-2\pi i n q)}{n^{m + 3/2}} =$$

$$= -\frac{\exp(-i\pi/4)}{\sqrt{2\pi}\, s} \sum_{1\ n}^{\infty} \frac{\exp(-2\pi i n q)}{n^{3/2}} + O[s^{-3}] \tag{2.10}$$

Expansion for small s and q:

$$U^{\pm}(s, q) = \frac{1}{2} \ln 2 + O[s] \tag{2.11}$$

$$U^{-}(s,\ q) = -\frac{1}{2}(\ln 2 - i\frac{\pi}{2}) + \ln(s + \sqrt{4\pi q}) + 0\ [s] \qquad (2.12)$$

Hankel Functions

$$\sigma_o(x) = -\frac{1}{\pi}\arg[H_o^{(2)}(x)] = \frac{x}{\pi} - \frac{1}{4} - \frac{1}{8\pi x} + \cdots \qquad (2.13)$$

$$\tau_1(x) = -\frac{1}{\pi}\arg[H_1^{(2)}(x)] + \frac{1}{2} = \frac{x}{\pi} + \frac{1}{4} + \frac{3}{8\pi x} - \cdots \qquad (2.14)$$

$$\sigma_m = -\frac{1}{\pi}\arg[H_m^{(2)}(x)] = \frac{1}{\pi}[x - \frac{2m+1}{4}\pi + \frac{m^2 - 1/4}{2x} \qquad (2.15)$$

$$\tau_m = -\frac{1}{\pi}\arg[H_m^{(2)}(x)] + \frac{1}{2} = \frac{1}{\pi}[x - \frac{2m+1}{4}\pi + \frac{m^2 + 3/4}{2x} - \cdots \qquad (2.16)$$

Integrals

$$2i\int_0^{\pi}\sin(x\cos\phi)\cos\phi\ d\phi = 2\pi i J_1(x) \qquad (2.17)$$

$$2\int_0^{\pi}\cos(x\cos\phi)\ d\phi = 2\pi J_o(x) \qquad (2.18)$$

$$\int_0^{\pi}[\exp(ix\cos\xi) + (-)^m\exp(-ix\cos\xi)]\ \exp(im\xi)\ d\xi = \int_0^{2\pi}\exp(ix\cos\xi)\exp(im\xi)d\xi$$

$$= 2\pi i^m J_m(x) \qquad (2.19)$$

$$\int_0^{\pi}[\exp(ix\cos\xi) - (-)^m\exp(-ix\cos\xi)]\ \exp(im\xi)\cos\xi d\xi =$$

$$\int_0^{2\pi}\exp(ix\cos\xi)\exp(im\xi)\cos\xi d\xi = 2\pi i^{m+1}\ \frac{J_{m+1}(x) - J_{m-1}(x)}{2} =$$

$$= -2\pi i^{m+1}\ J'_m\ (x) \qquad (2.20)$$

$$\int_0^{\pi} [\exp(ix\cos\xi) - (-)^m \exp(-ix\cos\xi)]\exp(im\xi)\sin\xi d\xi =$$

$$\int_0^{2\pi} \exp(ix\cos\xi)\exp(im\xi)\sin\xi d\xi = 2\pi i^m \frac{J_{m+1}(x) + J_{m-1}(x)}{2}$$

$$= 2\pi i^m \frac{mJ_m(x)}{x} \tag{2.21}$$

$$\int \cos(\frac{\pi\xi^2}{2a^2}y^2 - uy)dy = \frac{a}{\xi}\left[\cos\frac{u^2y^2}{2\pi\xi^2}\, C\,(\frac{\xi y}{a} - \frac{ua}{\pi\xi}) + \sin\frac{u^2y^2}{2\pi\xi^2}\, S\,(\frac{\xi y}{a} - \frac{ua}{\pi\xi})\right] \tag{2.22}$$

$$\int \sin(\frac{\pi\xi^2}{2a^2}y^2 - uy)dy = \frac{a}{\xi}\left[\cos\frac{u^2a^2}{2\pi\xi^2}\, S\,(\frac{\xi y}{a} - \frac{ua}{\pi\xi}) + \sin\frac{u^2a^2}{2\pi\xi^2}\, S\,(\frac{\xi y}{a} - \frac{ua}{\pi\xi})\right] \tag{2.23}$$

where $C(x)$, $S(x)$ are the Fresnel's integrals.

Fresnel integrals and auxiliary function

$$C(t) = \pm\left[\frac{1}{2} + f(t)\sin\frac{\pi t^2}{2} - g(t)\cos\frac{\pi t^2}{2}\right] \tag{2.24}$$

$$S(t) = \pm\left[\frac{1}{2} - f(t)\cos\frac{\pi t^2}{2} - g(t)\sin\frac{\pi t^2}{2}\right] \tag{2.25}$$

where $f(t)$ and $g(t)$ are the so called *auxiliary functions*, and plus and minus signs apply to positive and negative values of t respectively.

$$f'(t) = -\pi t g(t) \quad ; \quad g'(t) = \pi t f(t) - 1 \tag{2.26}$$

For $t \to 0+$

$$f(t) = \frac{1}{2} - \frac{\pi}{4}t^2 + \ldots \quad ; \quad g(t) = \frac{1}{2} - t + \ldots \tag{2.27}$$

For $t \to +\infty$

$$f(t) \simeq \frac{1}{\pi t} - \frac{1}{\pi^3 t^5} + \ldots \quad ; \quad g(t) \simeq \frac{1}{\pi t^3} - \frac{1}{\pi^4 t^7} + \ldots \tag{2.28}$$

$$f(-t) = f(t) \quad ; \quad g(-t) = g(t) \tag{2.29}$$

CHAPTER 3

WEINSTEIN'S SOLUTION FOR THE PARALLEL PLATE OPEN-ENDED WAVEGUIDE

The problem of radiation from an open-ended parallel plate waveguide has been rigorously solved by Weinstein in 1948, (results are referred to in [20], Ch.1), and independently by Heins [29], essentially by applying Wiener-Hopf techniques. Solutions to this problem (and also to the case of a circular waveguide) are referred to in a number of books [30-33]. Hereafter, Weinstein's solution is summarized(+) with reference to Fig. 3. In addition, only *far-fields* and $-\pi/2 \leq \phi \leq \pi/2$ (*front space*) are considered, where the latter condition can easily be relaxed.

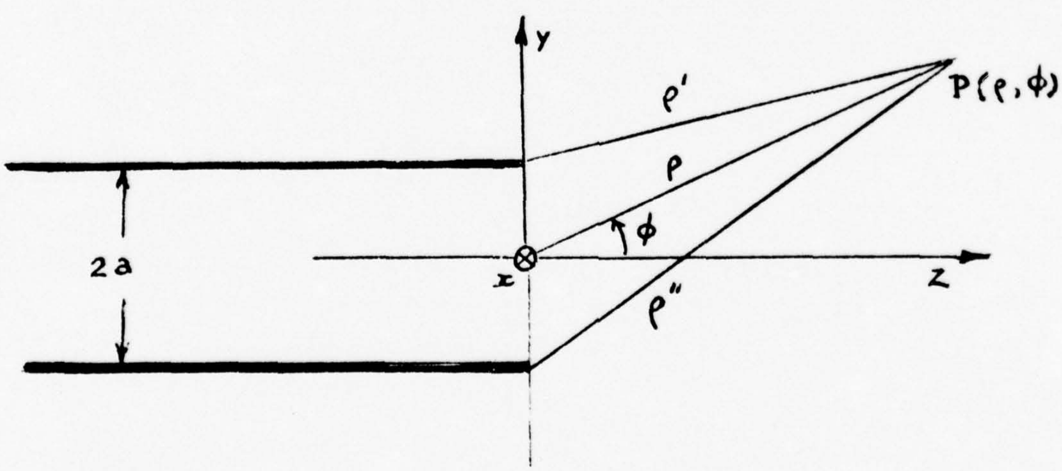

Fig. 3. Radiation from an open-ended parallel plate waveguide.

(+)Note that in the original Weinstein's solution [20] (i) the assumed time dependence is $\exp(-i\omega t)$, (ii) the z-axis is pointing toward the waveguide, (iii) the unit system is the gaussian one and (iv) symbols are slightly different from those here adopted.

For an incident TE_{no} wave (no x-variation) the fields can be derived, via (1.6-7), from a vector potential $\underline{A} = A\hat{x}$ where:

for n *odd:*

$$A = \frac{\zeta J_n}{\omega} \quad \frac{\exp[-i(\beta\rho + \pi/4)]}{\sqrt{2\pi\beta\rho}} \quad \frac{\cos\frac{\phi}{2}}{\cos\frac{\phi_n}{2}} \quad \frac{\cos(\beta a \sin\phi)}{\cos\phi - \cos\phi_n} \exp(U_n^+ - U^+) \tag{3.1}$$

for n *even:*

$$A = \frac{i\zeta J_n}{\omega} \quad \frac{\exp[-i(\beta\rho + \pi/4)]}{\sqrt{2\pi\beta\rho}} \quad \frac{\cos\frac{\phi}{2}}{\cos\frac{\phi_n}{2}} \quad \frac{\sin(\beta a \sin\phi)}{\cos\phi - \cos\phi_n} \exp(U_n^- - U^-) \tag{3.2}$$

In (3.1-2) the angle ϕ_n is related to the waveguide propagation constant β_n by:

$$\beta_n = \sqrt{\beta^2 - (n\pi/2a)^2} = \beta\cos\phi_n \quad ; \tag{3.3}$$

$\underline{J}_n = J_n\hat{x}$ in the *incident surface current* at $z = 0$ related to the incident electric field at $z = 0$, $\underline{E}_o = E_o\hat{x}$, by:

$$J_n = \frac{E_o \sin\phi_n}{j\zeta}, \quad \text{lower rim,} \tag{3.4}$$

$$J_n = \frac{(-)^{n+1} E_o \sin\phi_n}{j\zeta}, \text{ upper rim,} \tag{3.5}$$

$U^{\pm}(s, q)$ is the Weinstein diffraction function (2.7):

$$U^{\pm}(s, q) = U^{\pm}\left(\sqrt{2\beta a \cos\phi}, \frac{\beta a}{\pi}\right) , \tag{3.6}$$

and

$$U_n^{\pm}(s, q) = U^{\pm}\left(\sqrt{2\beta a \cos\phi_n}, \frac{\beta a}{\pi}\right) . \tag{3.7}$$

Results (3.1-2) are *approximated only* because the approximate expression (2.7) for $U^{\pm}$ has been used. They become exact if the exact expression (2.1) is adopted with $\sigma = \cos\phi$, $\rho = 2\beta a$. However, ever for moderate value of βa, the difference between the two formulations is negligible save in the neighbors of $|\phi| = \pi/2$, where it remains small.

For an incident TM_{no} wave, the fields can be derived from a vector potential $\underline{A} = A\hat{z}$ where:

for n *odd:*

$$A = \frac{\zeta J_n}{\omega} \quad \frac{\exp[-i(\beta\rho + \pi/4)]}{\sqrt{2\pi\beta\rho}} \quad \frac{\cos\frac{\phi_n}{2}}{\cos\frac{\phi}{2}} \quad \frac{\cos(\beta a \sin\phi)}{\cos\phi - \cos\phi_n} \exp(U_n^+ - U^+) \tag{3.8}$$

for n *even:*

$$A = \frac{i\zeta J_n}{\omega} \quad \frac{\exp[-i(\beta\rho + \pi/4)]}{\sqrt{2\pi\beta\rho}} \quad \frac{\cos\frac{\phi_n}{2}}{\cos\frac{\phi}{2}} \quad \frac{\sin(\beta a \sin\phi)}{\cos\phi - \cos\phi_n} \exp(U_n^- - U^-) \tag{3.9}$$

where $U^{\pm}$ is again given by (3.6) and the *incident surface current* $\underline{J}_n = J_n\hat{x}$ at $z = 0$ is related to the incident magnetic field at $z = 0$, $\underline{H} = H_o\hat{x}$, by:

$$J_n = H_o\,, \quad \text{upper rim} \tag{3.10}$$

$$J_n = (-)^{n+1} H\,, \quad \text{lower rim} \tag{3.11}$$

Solutions (3.1, 2, 8, 9) deserve a comment. For βa large, use of (2.10) shows that:

$$\exp[U_n^{\pm} - U^{\pm}] = 1 + O[1/\sqrt{\beta a}] \tag{3.12}$$

Accordingly, the left-hand side of (3.12) is an *interaction factor* which approaches unity when the spacing between the two plates is large.

Let us momentarily neglect this factor. It is immediately recognized that the fields are proportional to:

$$\frac{\exp[-i(\beta\rho + \pi/4)]}{\sqrt{2\pi\beta\rho}} \frac{\cos\frac{\phi}{2}}{\cos\frac{\phi_n}{2}} \frac{\exp(-i\beta\rho') \pm \exp(-i\beta\rho'')}{\cos\phi - \cos\phi_n} \tag{3.13}$$

for the TE case, and to

$$\frac{\exp[-i(\beta\rho + \pi/4)]}{\sqrt{2\pi\beta\rho}} \cos\frac{\phi_n}{2} \sin\frac{\phi}{2} \frac{\exp(i\beta\rho') \pm \exp(-i\beta\rho'')}{\cos\phi - \cos\phi_n} \tag{3.14}$$

for the TM case. In (3.13-14), the plus sign applies to n odd and the minus sign to n even. The total field is then obtained as the superposition of two cylindrical waves coming from the upper and lower rim of the guide respectively. When the expression for the incident current (3.4-5, 10-11) is taken into account, the similarity between (3.13-14) and the metal half-plane scattering problem [34] is immediately recognized. Each constituent component of (3.13-14) coincides with the asymptotic evaluation of the field scattered by a metal half-plane when the incident wave is plane, at an angle ϕ_n with the two rims respectively and with polarization $\underline{E} = E_o \hat{x}$ (TE case) or $\underline{H} = H_o \hat{x}$ (TM case). This is obviously consistent with the splitting of the incident field in a set of two plane waves at angles $\pm\phi_n$ with respect to the z-axis. The above similarity has been recognized from the beginning [20] and is the basis for all quasi-optical approaches to this problem. There is, however, a profound difference between the half-plane problem and the one at hand, and this has been so far apparently overlooked. *Each constituent term* in (3.13-14) *diverges* at $\phi = \phi_n$ (lit-shadow and lit-reflection boundaries respectively) while the *total field does not.* Consider, for instance, (3.1). When $\phi = \phi_n$ then $\beta a \sin\phi_n = n\pi/2$ and is the normalized eigenvalue of the parallel plate guide. Accordingly,

$$\lim_{\phi\to\phi_n} \frac{\cos(\beta a \sin\phi)}{\cos\phi - \cos\phi_n} = (i)^{n-1} \frac{\beta a \cos\phi_n}{\sin\phi_n} \tag{3.15}$$

and is finite. The reason for that is very basic and quite general. Consider the sketch of Fig. 4 where ray tracing is depicted for the parallel plate guide.

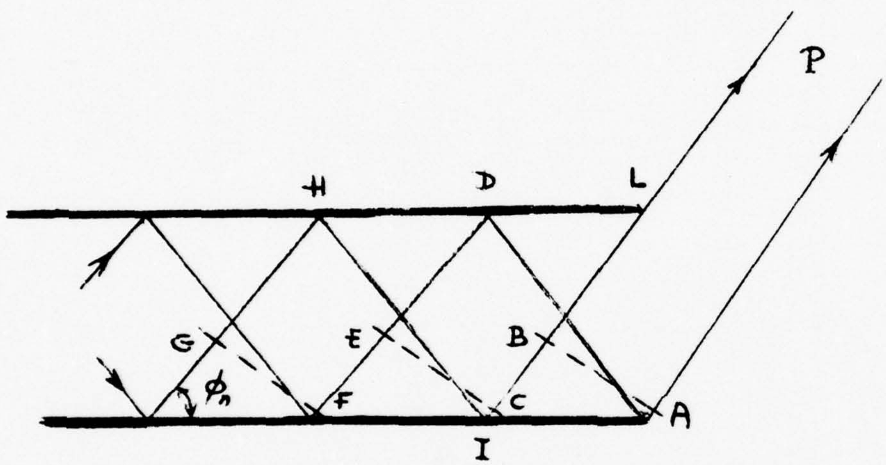

Fig. 4. Ray tracing inside a parallel-plate waveguide.

Points C and E, as well as G and F, are on an equiphase surface; accordingly, the phase delay along the two paths GHIC and FE should differ of an integer number of 2π (*consistency relationship*, the optical analog of field dispersion equation). Consider now the phase delay along the two paths CBP and EDAP. The phase introduced by the scattering process at A and L does not depend on the angle, so that it is equal for the two paths. However, the usal phase delay of π at A (due to the reflection) is now missed, since the reflection coefficient is substituted by a scattering coefficient. Accordingly, the two ray contributions at P from L and A are opposite in phase.

The following conclusion can be drawn. For the half-plane diffraction problem, the spectral components of the scattered field display pole singularity (in the wavenumber space) which approach the saddle point position at lit-shadow, lit-reflection, so that the steepest descent method evaluation (performed in the usual way) becomes invalid (resort should be made to the Pauli-Clemmow modified steepest descent method of integration [35]). On the contrary, these pole singularities cancel each other for the problem at hand, so that the usual asymptotic form for the scattered field is valid through all space.

The interaction factor (3.12) is now considered. Use of (2.10), valid for $2\beta a\cos\phi$ large, shows that:

$$\exp(U_n^{\pm} - U^{\pm}) = 1 \pm \frac{\exp[-i(2\beta a + \pi/4)]}{\sqrt{4\pi\beta a}} \frac{\cos\phi - \cos\phi_n}{\cos\phi\cos\phi_n} \sum_{0}^{\infty}{}_n (\pm)^n \frac{\exp(-i2n\beta a)}{(n+1)^{3/2}} + 0\,[1/\beta a] \tag{3.16}$$

When (3.16) is substituted into (3.1-2, 8-9) an extra term appears for the vector potential $\underline{A}$. This new term can be easily constructed using the model of interaction process depicted in Fig. 5 (*single interaction*). The ray along BC explains the term n = 0 of the series in (3.16); and the remaining terms of the series are due to the nonuniform illumination of the edge. For future reference it is important to recognized that $2\pi q = 2\beta a$ takes care of the phase delay of the ray path between the two edges.

When $2\beta a$ is small, use of (2.4-5) shows that:

$$\exp\left(U_n^+ - U^+\right) \simeq 1 \tag{3.17}$$

$$\exp\left(U_n^- - U^-\right) = \frac{\cos\frac{\phi_n}{2}}{\cos\frac{\phi}{2}} \tag{3.18}$$

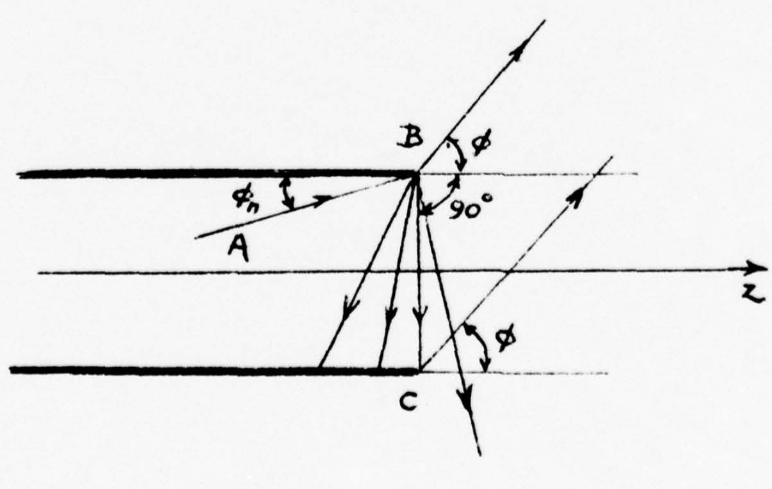

Fig. 5. Ray optical description of the interaction process.

CHAPTER 4

THE ELEMENTARY SCATTERING COEFFICIENTS

The field radiated by an open-ended parallel-plate waveguide has been represented, according to Weinstein's theory, as the superposition of *cylindrical waves* from the rims of the guide. The main idea is now to represent each cylindrical wave as superposition of *elementary spherical waves* radiated at each point along the rim. This goal can be accomplished by noting that, asymptotically as $\beta\rho \to \infty$,

$$-2\pi i F(0)\,\frac{\exp[-i(\beta\rho + \pi/4)]}{\sqrt{2\pi\beta\rho}} \simeq \int_{-\infty}^{+\infty} F(\xi)\,\frac{\exp(-i\beta\sqrt{\rho^2 + \xi^2})}{\sqrt{\rho^2 + \xi^2}}\,d\xi \qquad (4.1)$$

Here ρ is identified with ρ' (ρ'') in (3.1-2, 8-9), ξ with a curvilinear coordinate along the upper (lower) rim of the parallel-plate waveguide, $\sqrt{\rho^2 + \xi^2} = r$ is the radial distance from rim point to field point and F(0) can be immediately obtained by inspection from (3.1-2, 8-9).

The next step is the determination of $F(\xi)$, which should coincide with F(0) times an arbitrary factor which reduces to zero as $\xi \to 0$. This factor, in general, will not play an important role when applications are made to *large* apertures. As a matter of fact, superposition of elementary radiation from the rim of the aperture will result in an integral, and in the asymptotic evaluation of such integral the above factor is unimportant. Only the $(\cos\phi_n - \cos\phi)$ term appearing in the denominator of (3.1-2, 8-9) should be handled with care to avoid a singular behavior of the elementary scattering coefficients which are going to be introduced.

As a conclusion, the following *Ansatz* will be made.

Consider an open ended waveguide *uniform* along the z-axis. Choose two points Q, Q' on the rim of the waveguide such that

(i) the *tangent planes* to the waveguide walls at Q, Q' are *parallel*; their distance is 2a;

(ii) the *incident surface currents* $\underline{J}_{\parallel}$, $\underline{J}_{\perp}$, parallel and perpendicular to the rim, have equal amplitude at (Q, Q') and either equal or opposite sign.

Then, with reference to Fig. 6, the following elementary vector potentials are associated to the elementary currents $d\underline{J}_{\parallel} = \underline{J}_{\parallel}ds$, $d\underline{J}_{\perp} = \underline{J}_{\perp}ds$ respectively:

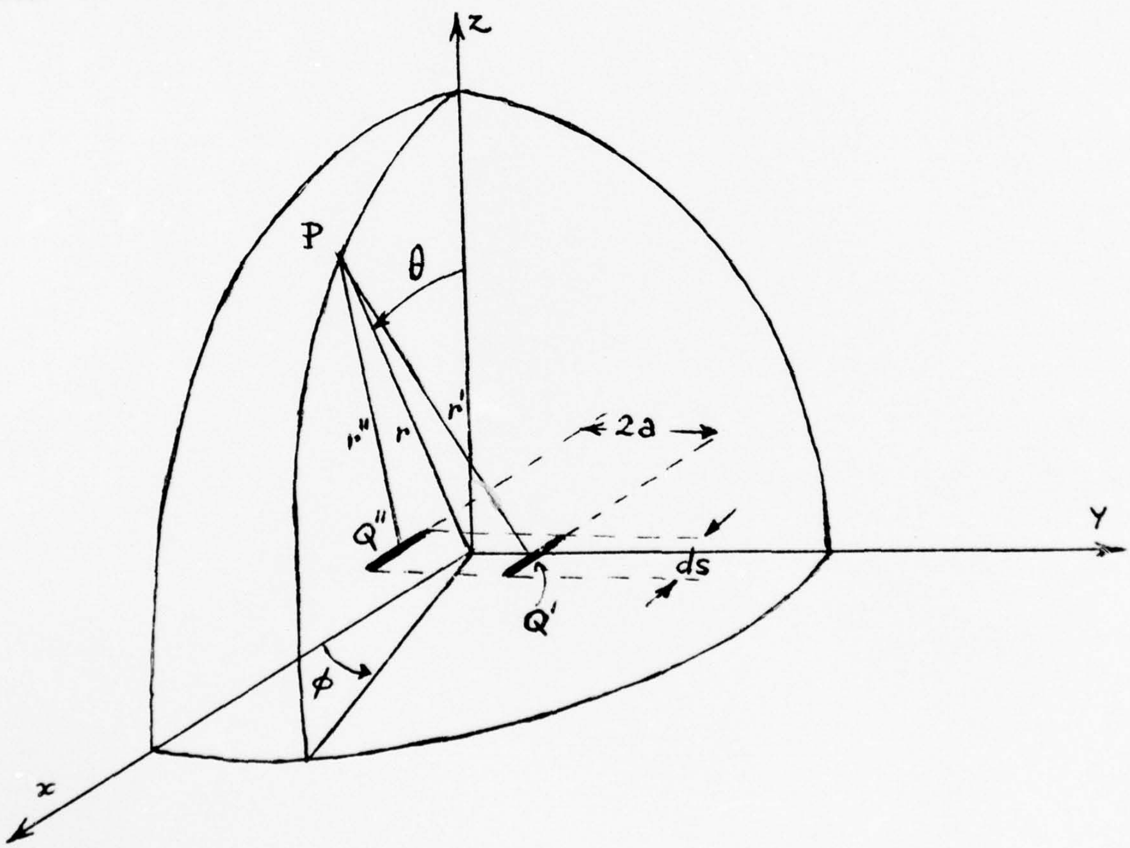

Fig. 6. Geometry of elementary radiation.

$$d\underline{A} = \zeta \frac{dJ_{\parallel}}{-i\omega} \frac{\exp(-i\beta r)}{2\pi r} \frac{\cos\frac{\Theta}{2}}{\cos\frac{\Theta_g}{2}} \frac{\exp(i\beta a\sin\Theta\sin\phi) \pm \exp(-i\beta a\sin\Theta\sin\phi)}{\sqrt{1-\sin^2\Theta\sin^2\phi} - \cos\Theta_g} \cdot \exp(U_g^{\pm} - U^{\pm}) \qquad (4.2)$$

$$d\underline{A} = \zeta \frac{dJ_{\perp}}{-i\omega} \frac{\exp(-i\beta r)}{2\pi r} \frac{\cos\frac{\Theta_g}{2}}{\cos\frac{\Theta}{2}} \frac{\exp(i\beta a\sin\Theta\sin\phi) \pm \exp(-i\beta a\sin\Theta\sin\phi)}{\sqrt{1-\sin^2\Theta\sin^2\phi} - \cos\Theta_g} \cdot \exp(U_g^{\pm} - U^{\pm}) \qquad (4.3)$$

In (4.2-3) the choice of plus or minus sign is related to equal or opposite sign for the incident surface current at Q', Q" respectively; the angle Θ_g is given by $\beta_g = \beta\cos\Theta_g$, where β_g is the waveguide propagation constant. Note that $2\beta a\sin\Theta\sin\phi = |\underline{r}' - \underline{r}''|$ and that the elementary vector potentials (4.2-3) are everywhere finite provided that the dispersion relation

$$\exp(i\beta a\sin\Theta_g) \pm \exp(-i\beta a\sin\Theta_g) = 0 \qquad (4.4)$$

is valid.

It is obvious that application of (4.2-3) to the case of a parallel plate waveguide using the asymptotic evaluation (4.1) of the superposition integral gives (3.1-2) and (3.8-9), respectively.

CHAPTER 5

RADIATION FROM OPEN ENDED CIRCULAR WAVEGUIDE. AZYMUTHALLY SYMMETRIC MODES.

Consider now an open ended circular waveguide as depicted in Fig. 7. The guide is excited by ϕ - independent modes (*azymuthally symmetric modes*), an assumption which will be relaxed in Sect. 6.

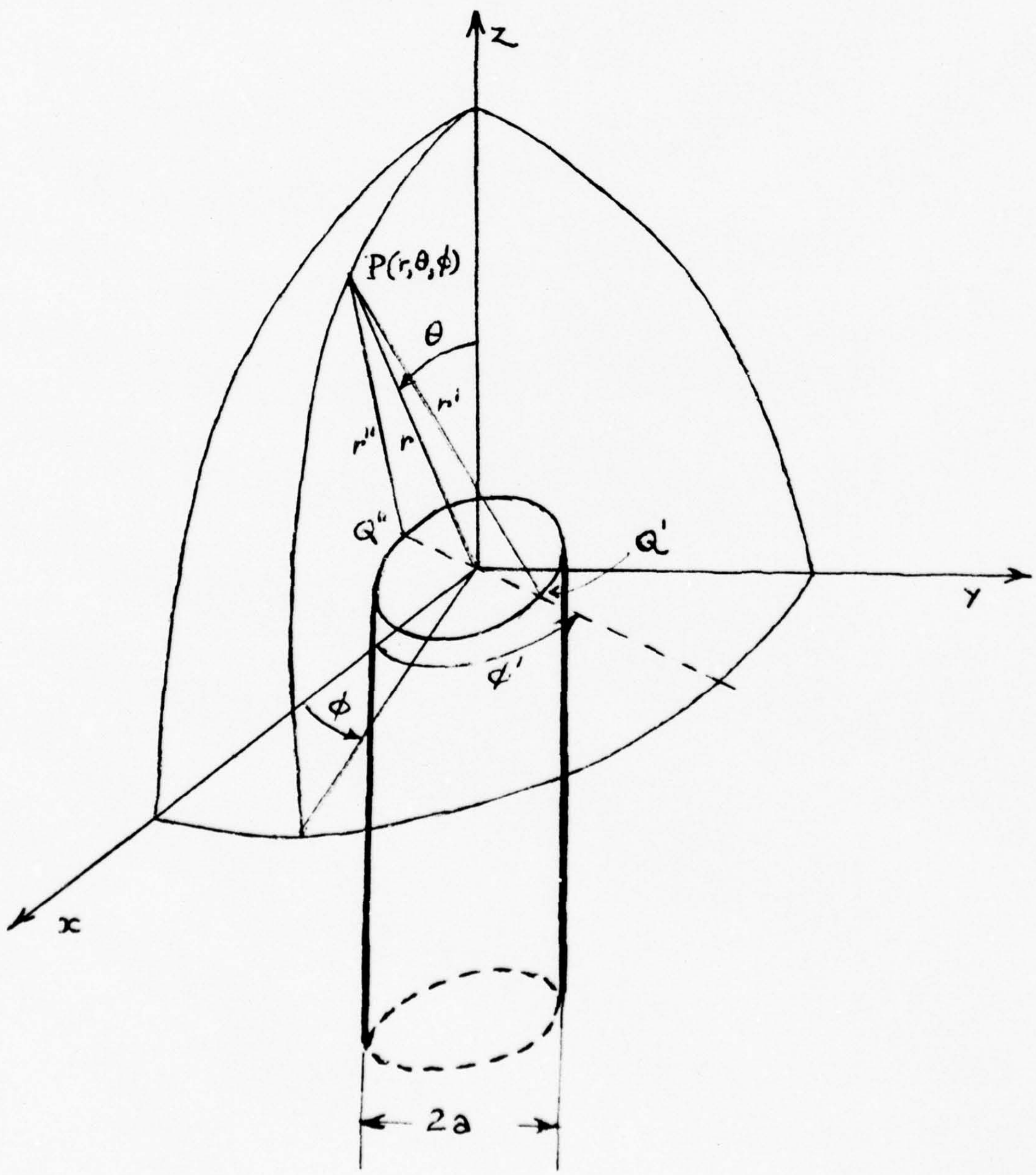

Fig. 7. Radiation from an open-ended circular waveguide.

For a TM_{no} incident mode the surface current incident on the rim has only a z-component, $\underline{J} = J_z \hat{z}$; the dispersion relation is:

$$J_o(k_t a) = 0 \quad ; \quad k_t = \beta \sin\Theta_g \; ; \tag{5.1}$$

the distances from Q', Q" to P are given by

$$r' \simeq r - a\sin\Theta\cos(\phi' - \phi) \quad ; \quad r'' \simeq r + a\sin\Theta\cos(\phi' - \phi) \; ; \tag{5.2}$$

the appropriate elementary scattering coefficient is (4.3) with the *plus* sign, since surface currents have the same sign at Q', Q". In conclusion, the total vector potential $\underline{A} = A\hat{z}$ will be proportional to the following integral:

$$\int_{\phi}^{\phi+\pi} \frac{\cos[\beta a\sin\Theta\cos(\phi' - \phi)]}{\sqrt{1 - \sin^2\Theta\cos^2(\phi' - \phi)} - \cos\Theta_g} \, d\phi' . \tag{5.3}$$

When $\beta a\sin\Theta$ is large the main contribution to the integral (5.3) comes from integration points close to $\phi' = \phi$. When $\sin\Theta \ll 1/\beta a$ and βa is large, the cosine term in the denominator of (5.3) does not play a significant role. Accordingly, we can put $\phi' = \phi$ *only in the denominator* of (5.3) and evaluate the integral (5.3) with the aid of (2.18). When all terms are taken into account, the final expression for the vector potential is:

$$A = - \frac{\zeta J_z a}{-i\omega} \; \frac{\exp(-i\beta r)}{2r} \; \frac{\cos\frac{\Theta_g}{2}}{\cos\frac{\Theta}{2}} \; \frac{J_o(\beta a\sin\Theta)}{\cos\Theta - \cos\Theta_g} \; \exp(U_g^+ - U^+) \tag{5.4}$$

Note that the vector potential is *everywhere finite* in view of the dispersion relation (5.1).

We still need to specify the values of s and q which appear in the Weinstein function U^+ (2.7). It has already been noted in Sec. 3 that $2\pi q$ should equal the phase delay between interacting points on the edge, such as Q' and Q". In this case of circular symmetry, the line joining Q' and Q" intersects the axis of the guide, which is a caustic line. Accordingly, the usual phase delay of $\pi/4$ should be added, so that the values of $U^{\pm}$ to be inserted into (5.4) are the following:

$$U^{+}(s,\ q) = U^{+}\left(\sqrt{2\beta a}\,\cos\Theta,\ \frac{\beta a - \pi/4}{\pi}\right) \tag{5.5}$$

$$U_g^{+}(s,\ q) = U^{+}\left(\sqrt{2\beta a}\,\cos\Theta_g,\ \frac{\beta a - \pi/4}{\pi}\right) \tag{5.6}$$

Expression (5.4) *coincides* with the exact solution to the problem [20], with a minor modification in the q parameter of the function U^{+}. According to Weinstein, the proper value of q to be inserted into (5.5-6) is:

$$q = \frac{1}{\pi}\arg[H_o^{(2)}(\beta a)] \tag{5.7}$$

while in (5.5-6) we have:

$$q = \frac{\beta a - \pi\ 4}{\pi} \tag{5.8}$$

Use of (2.13) shows that the difference between (5.7) and (5.8) is of order $1/8\pi\beta a$.

The TE_{no} incident case can be similarly treated. The surface current on the rim has only a ϕ component, $\underline{J} = J_\phi\,\hat{\phi}$; the dispersion relation is:

$$J_1(k_t a) = 0\ ,\quad k_t = \beta\sin\Theta_g \tag{5.9}$$

The appropriate elementary scattering coefficient is (4.2) with the *minus* sign, since surface currents have opposite signs at Q' and Q". The elementary vector potential $d\underline{A}$ due to the two rim elements $ad\phi$ at Q', Q" can be resolved into two components:

$$d\underline{A} = dA\cos(\phi' - \phi)\hat{\phi} - dA\sin(\phi' - \phi)\hat{\phi} \times \hat{z}, \tag{5.10}$$

parallel and normal to $\hat{\phi}$, respectively. It is readily seen that the integral of the latter component is equal to zero, while the integral of the former gives for the total vector potential $\underline{A} = A\hat{\phi}$:

$$A = -i\zeta \frac{J_\phi a}{-i\omega} \frac{\exp(-i\beta r)}{2r} \frac{\cos\frac{\theta}{2}}{\cos\frac{\theta_g}{2}} \frac{J_1(\beta a \sin\theta)}{\cos\theta - \cos\theta_g} \exp(U_g - U^-) \qquad (5.11)$$

Note that the vector potential is everywhere finite due to the dispersion relation (5.9). The function U should be computed for the same value (5.8) of q.

Expression (5.11) *coincides* with the exact solution to the problem [20], with the only minor modification:

$$q = -\frac{1}{\pi} \arg[H_1^{(2)}(\beta a)] \qquad (5.12)$$

Use of (2.14) shows that the difference between (5.12) and (5.8) is of order $3/8\pi\beta a$.

CHAPTER 6

RADIATION FROM OPEN-ENDED CIRCULAR WAVEGUIDES CONTINUED: AZYMUTHALLY ASYMMETRIC MODES

Radiation from an open-ended circular waveguide is now considered with TE_{nm} or TM_{nm} excitation, wherein the ϕ-dependence of the fields inside the waveguide is of type $\exp(im\phi)$. The analysis is more involved, although the procedure parallels that for TE_{no} or TM_{no} excitation.

First, TE_{nm} mode excitation is considered, wherein the surface current incident on the rim, $\underline{J}\exp(im\phi)$, is:

$$\underline{J} = J_\phi\hat{\phi} + J_z\hat{z} \quad , \quad J_z = -\frac{m\cos\Theta_g}{\beta a\sin^2\Theta_g} J_\phi \quad , \tag{6.1}$$

and the dispersion relation is:

$$J'_m(k_t a) = 0 \quad , \quad k_t = \beta\sin\Theta_g . \tag{6.2}$$

The component J_z of the incident surface current produces a vector potential, $\underline{A}_1 = A_1\hat{z}$, which can be computed by using (4.3), (5.2) and (2.19):

$$A_1 = i^m \frac{\zeta J_z a}{-i\omega} \frac{\exp(-i\beta r)}{2r} \frac{\cos\frac{\Theta_g}{2}}{\cos\frac{\Theta}{2}} \exp(im\phi) \frac{J_m(\beta a\sin\Theta)}{\cos\Theta - \cos\Theta_g} \exp(U_g^+ - U^+) \tag{6.3}$$

where

$$U^+(s,\ q) = U^+(\sqrt{2\beta a}\cos\Theta,\ \frac{\beta a - \pi/4 - m\pi/2}{\pi}) \tag{6.4}$$

Note the expression of q in (6.4). The term $-\pi/4$ accounts for the phase delay at the caustic line crossing on the axis of the guide. The extra term $m\pi/2$ takes into account the equal (m even) or opposite (m odd) sign of J_z

at Q', Q" (see Fig. 7), as follows upon use of (2.8-9).

For the vector potential $\underline{A}_2 = A_2'\hat{\phi} - A_2''\hat{\phi} \times \hat{z}$ associated to the component J_ϕ we have, upon use of (4.2), (5.2) and (2.20-21):

$$A_2' = -i^{m+1} \frac{\zeta J_\phi a}{-i\omega} \frac{\exp(-i\beta r)}{2r} \frac{\cos\frac{\Theta}{2}}{\cos\frac{\Theta_g}{2}} \exp(im\phi) \frac{J_m'(\beta a \sin\Theta)}{\cos\Theta - \cos\Theta_g} \exp(U_g^- - U^-) \tag{6.5}$$

$$A_2'' = -i^m \frac{\zeta J_\phi a}{-i\omega} \frac{\exp(-i\beta r)}{2r} \frac{\cos\frac{\Theta}{2}}{\cos\frac{\Theta_g}{2}} \exp(im\phi) \frac{m}{\beta a \sin\Theta} \frac{J_m(\beta a \sin\Theta)}{\cos\Theta - \cos\Theta_g} \exp(U_g^- - U^-) \tag{6.6}$$

$$U^-(s, q) = U^-\left(\sqrt{2\beta a}\cos\Theta, \frac{\beta a - \pi/4 - m\pi/2}{\pi}\right) \tag{6.7}$$

The transverse components of the vector potential A_Θ and A_ϕ, are given by:

$$A_\phi = A_2' \quad ; \quad A_\Theta = A_2' \cos\Theta - A_1 \sin\Theta \tag{6.8}$$

$$A_\phi = -i^{m+1}\zeta \frac{J_\phi a}{-i\omega} \frac{\exp(-i\beta r)}{2r} \frac{\cos\frac{\Theta}{2}}{\cos\frac{\Theta_g}{2}} \exp(im\phi) \frac{J_m'(\beta a \sin\Theta)}{\cos\Theta - \cos\Theta_g} \exp(U_g^- - U^-) \tag{6.9}$$

$$\Delta_\Theta = -i^m\zeta \frac{J_\phi a}{-i\omega} \frac{\exp(-i\beta r)}{2r} \frac{2\cos\frac{\Theta}{2}}{\sin\frac{\Theta_g}{2}} \exp(im\phi) \frac{J_m(\beta a \sin\Theta)}{\sin\Theta \sin\Theta_g} \Delta \tag{6.10}$$

$$\Delta = -\frac{im}{2\beta a} \tag{6.11}$$

where U^- is given by (6.7), use has been made of (6.1) and only terms up to order Δ have been retained in (6.10).

Expressions (6.9-10) are now compared to the expansion of exact results [20] up to terms of order Δ^2 included. The ϕ-component of the exact vector potential equals (6.9) times

$$1 + \left(\frac{m}{k_t a}\right)^2 \frac{1 + \cos\Theta_g}{1 + \cos\Theta} \frac{\cos\Theta - \cos\Theta_g}{2} \qquad (6.1?)$$

provided that in (6.7):

$$q = \frac{1}{\pi} \arg H_m^{(2)\prime}(\beta a) \qquad (6.12)$$

The Θ-component of the vector potential equals (6.10) times:

$$1 - \Delta^2 = 1 + \left(\frac{m}{k_t a}\right)^2 \cos^2\Theta_g \qquad (6.13)$$

Use of (2.16) shows that the difference between the two values of q is small provided that βa is even moderately large compared to unity. The conclusion is drawn that *use of elementary scattering coefficients* (4.2-3) *leads to good results provided that* $k_t a > m$ *and the excitation is TE.*

Let us now turn to TM_{nm} modes, for which the incident surface current $\underline{J} \exp(im\phi)$ has only a z- component, $\underline{J} = J\hat{z}$, and the dispersion equation is:

$$J_m(k_t a) = 0, \qquad k_t = \beta \sin\Theta_g \qquad (6.14)$$

Then, use of (4.3), (5.2) and (2.19) will result in a vector potential $\underline{A} = A\hat{z}$ given by (6.3):

$$A = A_1 \qquad (6.15)$$

This vector potential will provide (E_Θ, H_ϕ) fields only, according to (1.6-7), while the exact solution [20] provides (E_Θ, H_ϕ) and (E_ϕ, H_Θ) fields as well. Comparison only between electric fields is necessary, since $H = E/\zeta$.

The exact expression for the E_Θ field, up to terms Δ^2 included, is the following:

$$E_\Theta = -i^m \zeta J_z a \frac{\exp(i\beta r)}{2r} \exp(im\phi) \exp(U_g^+ - U^+) \cdot$$

$$\cdot \left[\frac{\cos\frac{\Theta_g}{2}}{\cos\frac{\Theta}{2}} \frac{J_m(\beta a \sin\Theta)}{\cos\Theta - \cos\Theta_g} \sin\Theta - \frac{\cos\frac{\Theta}{2}}{2\cos\frac{\Theta_g}{2}} \frac{m J_m(\beta a \sin\Theta)}{\beta a \sin\Theta} \frac{m}{\beta a} \right] \qquad (6.16)$$

where the argument q of the function U^+ is given by :

$$q = \frac{1}{\pi} \arg[H_m^{(2)}(\beta a)] \tag{6.17}$$

As for the TE case, difference in the two values of q is negligible as shown by (2.15).

The first term in (6.16) is *exactly predicted. The second term, which is of order* $(m/\beta a)^2$, *is not predicted* at all. This term is important only for m = 1 since , at variance with the other one, is different from zero for $\Theta = 0$. However, the ratio of the first term to the second equals:

$$\left(\frac{2\beta a}{m}\right)^2 \frac{\Theta^2}{tg^2 \frac{\Theta_g}{2}} \tag{6.18}$$

for small values of Θ, so that the first term dominates for

$$\Theta > \frac{m}{2\beta a}\, tg\, \frac{\Theta_g}{2} \tag{6.19}$$

The exact expression for the *unpredicted* ϕ-component of electric field is, up to terms of order Δ,

$$E_\phi = i^m \zeta J_z a \; \frac{\exp(-i\beta r)}{2r} \exp(im\phi) \; \frac{J_m'(\beta a \sin\Theta)}{\cos\frac{\Theta_g}{2} \cos\frac{\Theta}{2}} \Delta \tag{6.20}$$

The conclusion is drawn that, *for* TM_{nm} *excitation, use of elementary scattering coefficients (4.2-3) leads to good results provided that* $\beta a/m >> 1$; discrepancies with the exact expressions for the fields arise only in a very small angular region centered at $\Theta = 0$, for m = 1; and, in the case of linearly polarized excitation, for the cross-polarized component of the field. A way to overcome these inconveniences is given in section 9.

CHAPTER 7

RADIATION FROM OTHER OPEN-ENDED GUIDING STRUCTURES

Some other open-ended guiding structures will now be considered briefly.

The vector potential associated to an open-ended *coaxial cable* of outer and inner diameters 2a,2b respectively can be easily computed using (4.3) and $\Theta_g = 0$ for TEM excitation. The final result for $\underline{A} = A\hat{z}$ is the following:

$$A = -\frac{\zeta I}{-i\omega}\frac{e^{-i\beta r}}{2\pi r}\cos\frac{\Theta}{2}\;\frac{J_o(\beta a\sin\Theta) - J_o(\beta b\sin\Theta)}{\sin^2\Theta}\exp(U_o^- - U^-) \qquad (7.1)$$

$$U^-(s,\ q) = U^-\left(\sqrt{\beta(b-a)}\cos\Theta,\ \frac{\beta(b-a)}{2\pi}\right) \qquad (7.2)$$

Note that Θ is the angle with the cable axis, I is the total incident current and only interaction between inner and outer walls has been considered. When $\beta a \ll 1$ (*small cable*), eq. (7.1) transforms into:

$$A = -\frac{\zeta I}{-i\omega}\frac{\exp(-i\beta r)}{8\pi^2 r}\beta^2 S = \frac{\mu}{4\pi}\frac{\exp(-i\beta r)}{r}\left[\frac{-i\omega\sqrt{\varepsilon\mu}S\ I}{2\pi}\right] \qquad (7.3)$$

where $S = \pi(b^2 - a^2)$ in the aperture area and use has been made of (3.18).

Equation (7.3) shows that the cable aperture is equivalent to a z-directed electric dipole of moment $\sqrt{\varepsilon\mu}SI/2\pi$. This is the *exact* value which is computed assuming only the electric field (twice as big as the incident one) to exist across the aperture and then applying the equivalence theorem [36].

For an open-ended *strip line* of width 2b and conductor spacing 2a, the E-plane field can be easily computed using (4.3), (1.6) and $\Theta_g = 0$ (TEM excitation). The final result is the following:

$$\begin{cases} E_\Theta = -i\zeta \dfrac{2I}{r} \exp(-i\beta r) \cos\dfrac{\Theta}{2} \dfrac{\sin(\beta a \sin\Theta)}{\sin\Theta} \exp(U_o^- - U^-) & (7.4) \\ H_\phi = E_\Theta / \zeta & (7.5) \end{cases}$$

$$U^-(s, q) = U^-(\sqrt{2\beta a} \cos\Theta, \frac{\beta a}{\pi}) \qquad (7.6)$$

where I is the total incident current. Expressions (7.4-5) are consistent, in the low frequency case, with application of the equivalence theorem, as in the case of the coaxial cable.

Equations (7.4-5) do not depend on the strip width. Accordingly they are valid also for an open-ended *two-wire transmission line.*

The radiation from an open-ended rectangular waveguide can also be computed. Results are in good agreement with experimental data. They are not quoted hereafter only because some further assumption is necessary in the scattering coefficients (4.2-3).

The coaxial cable, the strip and the two-wire transmission line are non-dispensive guiding structures, when excited under TEM conditions. Accordingly, a pulse is not distorted while propagating along these guides; they are the simplest suitable *pulse radiators.* Transient radiation from these configurations has been given in [26, 36] neglecting the interaction factor. Fourier inversion of this factor in the time domain is necessary for a complete analysis of transient radiation properties of the structures previously analyzed.

CHAPTER 8

THE INCLUSION OF A FLARE ANGLE

For the applications viewpoint, it is highly desirable to have scattering coefficients which can be applied to *flared* open-ended guiding structures, such as a horn. Unfortunately, no exact solution exists for radiation from a flared parallel-plate waveguide. However, there is a way to bypass this difficulty.

Consider the sketch of Fig. 8 where the terminal section of a flared parallel-plate waveguide is considered; let α be the flare angle. The field distribution on the mouth $z = 0$ of the flared guide will be different from that of an unflared parallel-plate guide with the same mouth dimension 2a at $z = 0$. For small flare angles, however, this difference is just a *phase error* due to a change in path length:

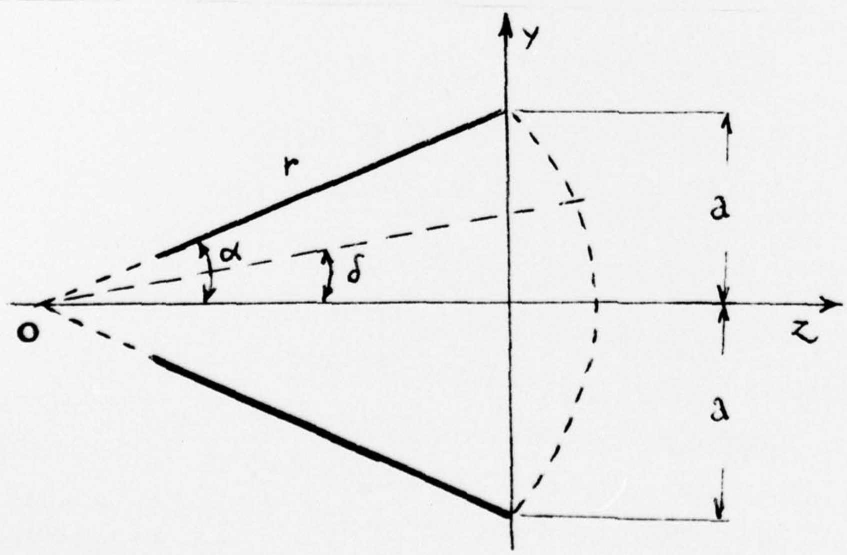

Fig. 8. Radiation from open-ended flared parallel-plate waveguide.

$$\Delta = r - \frac{r\cos\alpha}{\cos\delta} = \frac{a}{|\sin\alpha|} - \frac{a\cos\alpha}{|\sin\alpha|\cos\delta} = \frac{a}{|\sin\alpha|}\left[1 - \sqrt{\cos^2\alpha + \left(\frac{y}{a}\right)^2 \sin^2\alpha}\right]$$

$$\simeq \frac{a|\sin\alpha|}{\alpha}\left(1 - \frac{y^2}{a^2}\right) \qquad (8.1)$$

where the only assuption used in the last derivation is that $\sin^2\alpha << 1$.

Accordingly, the phase error across the mouth will be:

$$\Delta\phi \simeq \frac{\pi}{2}\,\xi^2\left(1 - \frac{y^2}{a^2}\right), \quad \xi^2 = \frac{\beta_g\, a\,|\sin\alpha|}{\pi} \qquad (8.2)$$

where β_g is the propagation constant in the flared guide and ξ is a parameter describing the flare.

The field in the half-space $z > 0$ can be represented as the inverse Fourier transform (in wavenumber space) of the field distribution in the plane $z = 0$. We assume this field distribution to be coincident with the field distribution relative to an *unflared* parallel plate guide (of mouth dimension 2a) *times a phase error:*

$$\Delta\phi = \frac{\pi}{2}\xi^2\left(1 - \frac{y^2}{a^2}\right), \quad -a \leq x \leq a \qquad (8.3)$$

$$\Delta\phi = 0 \qquad |x| > a \qquad (8.4)$$

Accordingly, the field for $z > 0$ and, in particular, the far field, will be obtained as the convolution of the (known) field radiated by the (unflared) parallel plate waveguide and the *Fourier transform the complex exponential of* (8.3-4):

$$\psi = \frac{1}{2\pi}\int_{-\infty}^{-a} \exp(iuy)dy + \frac{1}{2\pi}\int_{-a}^{a} \exp(iuy)\exp\left[i\frac{\pi}{2}\xi^2\left(1 - \frac{y^2}{a^2}\right)\right] dy$$

$$+ \frac{1}{2}\int_{a}^{\infty} \exp(iuy)dy \qquad (8.5)$$

where u is the y-component of spatial frequency and equals $\beta\sin\phi$ (see Fig. 3) in real space.

Computation of (8.5) is rather involved and requires the use of (2.22-25). The final result is the following:

$$\psi = \delta(u) - \frac{a}{\pi}\left\{\frac{\sin ua}{ua} - \frac{1}{2i\xi}\left[f\left(\frac{ua}{\pi\xi} - \xi\right)\exp(iua) - f\left(\frac{ua}{\pi\xi} + \xi\right)\exp(-iua)\right]\right.$$

$$\left. - \frac{1}{2\xi}\left[g\left(\frac{ua}{\pi\xi} - \xi\right)\exp(iua) - g\left(\frac{ua}{\pi\xi} + \xi\right)\exp(-iua)\right]\right\} \qquad (8.6)$$

When ξ is small eq. (8.6) can be simplified by expanding the auxiliary functions f and g in the neighbors of $ua/\pi\xi$. Using (2.26) we get:

$$\psi = \delta(u) - \frac{a}{\pi}\left[\frac{\sin ua}{ua} - \cos ua\right]\left[1 - \frac{ua}{\xi}f\left(\frac{ua}{\pi\xi}\right) - i\,\frac{ua}{\xi}g\left(\frac{ua}{\pi\xi}\right)\right] = \delta(u) + \tilde{\psi}(u) \qquad (8.7)$$

As a check, it can be noted that, for $\xi \to 0$, the last bracketed term in (8.7) reduces to $-i(\pi\xi/ua)^2$, upon use of (2.28). Accordingly, the second term of (8.7) is proportional to ξ^2 and ψ reduces to a δ - function for $\xi = 0$, as it should be.

The new vector potential $\tilde{A}$, relative to the flared parallel plate wave-guide, is given by:

$$\tilde{A}(u) = \int_{-\infty}^{+\infty}\psi(u')\,A(u-u')du' = A(u) + \int_{-\infty}^{+\infty}\tilde{\psi}(u')A(u-u')du' \qquad (8.8)$$

where the vector potential A relative to the unflared parallel plate system is given by (3.1-2) and (3.8-9).

Due to the discontinuous behavior (2.24-25) of the function f(t) and g(t), the range of the integral which appears in (8.8) should be split into the two parts $(-\infty, 0)$ and $(0, \infty)$. For the latter we have, upon repeated integration by parts and use of (2.26):

$$\int_0^\infty \tilde{\psi}(u')A(u - u')du' = -\xi^2 \int_0^\infty \left[\frac{t}{\pi\xi} g\left(\frac{t}{\pi\xi}\right) - i \frac{t}{\pi\xi} f\left(\frac{t}{\pi\xi}\right)\right] \frac{dA(ua - t)}{dt} \frac{d}{dt} \frac{\sin t}{t} dt$$

$$-\xi^2 \int_0^\infty \left[\frac{t}{\pi\xi} g\left(\frac{t}{\pi\xi}\right) - i \frac{t}{\pi\xi} f\left(\frac{t}{\pi\xi}\right)\right] A(ua - t) \frac{1}{t} \frac{d}{dt} t \frac{d}{dt} \frac{\sin t}{t} dt \qquad (8.9)$$

Now, the bracketed term approaches -i and remains constant as $t > \pi\xi$ as follows upon use of (2.28); there is no serious error in taking this term approximately constant. Then, integration by parts of the first integral at right hand side of (8.9) gives:

$$\int_0^\infty \tilde{\psi}(u')A(u - u')du' = +i\xi^2 \frac{dA}{du} + i\xi^2 \int_0^\infty \left[A \frac{1}{t} \frac{d}{dt} t \frac{d}{dt} \frac{\sin t}{t} - \frac{dA}{dt} \frac{\sin t}{t}\right] dt \qquad (8.10)$$

Now the rapidly varying part of $A(ua - t)$ is of type $\exp(\pm it)$, when the aperture is large. Accordingly, $d^2A/dt^2 \simeq -A$ and (8.10) transforms into:

$$\int_0^\infty \tilde{\psi}(u')A(u - u')du' = +i\xi^2 \frac{dA}{du} + i\xi^2 \int_0^\infty \frac{\sin t - t \cos t}{t^3} A(ua - t)dt \qquad (8.11)$$

When also the remaining range of the integral (8.8) is taken into account, we get:

$$\int_0^\infty \tilde{\psi}(u')A(u - u')du' = + 2i\xi^2 \frac{dA}{du} + i\xi^2 \int_0^\infty \frac{\sin t - t \cos t}{t^3} \left[A(ua - t) - A(ua + t)\right] dt \qquad (8.12)$$

Eq. (8.12) provides a way for computing the scattering coefficients analogous to (4.2-3) and appropriate to the open ended flared waveguide.

CHAPTER 9

CONCLUDING REMARKS

A new formulation has been presented for radiation from open-ended guiding structures. It is essentially a field theory which calculates the field as an integral over the rim of the guide. No problems of ray tracing, or boundaries, or caustics arise. The theory can even be extended to open guides with a terminal flare angle. Although presented only for the front space, the formulation can be extended to the rear space.

The key point of the theory is the canonical problem of a parallel-plate waveguide. It is not unexpected that when the theory is applied to structures far from the canonical one, some discrepancies are found. This is, for example, the case for a circular waveguide excited by azymuthally asymmetric modes, where the incident field on the rim of the guide presents a ϕ-variation with no counterpart in the original canonical problem of a parallel plate waveguide. Obviously, we can take as *basic canonical problem that of the circular waveguide with azymuthally asymmetric modes;* and derive *more general*, although more complicated, *scattering coefficients* which would replace those of eqs. (4.2-3). In such a way the applicability of this new formulation would certainly be enlarged at the expense of some formal complications.

Guiding structures which are worth analyzing are the corrugated circular waveguide and the rectangular waveguide.

For the former, the different type of boundary conditions requires the use of incident *electric* as well as *magnetic* surface currents. A first rough way to handle the problem would be the use of scattering coefficients for the magnetic currents obtained from those of the electric one by duality. A better approach would be to use the solution of scattering by a half-plane with two different surface impedances (in the particularly suitable form [37]) to model new scattering coefficients.

For the latter, the formulation just presented for the open-ended parallel-plate waveguide is already able to give good results. However, the use of *different* dispersion relations for the E and H planes is necessary. This is certainly a rather unsatisfactory point and is again due to the necessity of handling a structure too different from the canonical one.

We believe that the limitations of this theory are *not* intrinsic but rather reflect its early stages. It is probably worth developing further this novel formulation, which presents a simplicity and a number of attractive features not available in current theories of high frequency radiation.

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